How to evaluate your z-score?

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Contents

- Understanding z-scores
- Multiple z-score analysis
- Evaluating z-scores in pairs
- Evaluating z-score history

Understanding z-scores

Normal Distribution

- The z-score has its origin to the Normal distribution, a symmetric bell shaped curve (also known as Gaussian) that plays a major role in statistics. It is denoted as $X \sim N\left(\mu, \sigma^2\right)$, with μ being its mean and σ its standard deviation.

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- For $x \in (-\infty, +\infty)$

$$f(x|\mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}$$

$$E[X] = \mu \qquad Var[X] = \sigma^2$$

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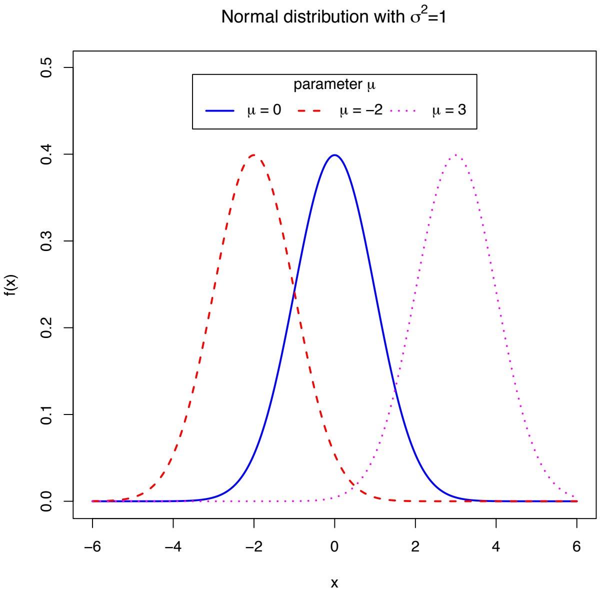
- The $Z \sim N(0, 1)$ is known as the "Standard" Normal Distribution and is related to the concept of z-scores

Normal Distribution



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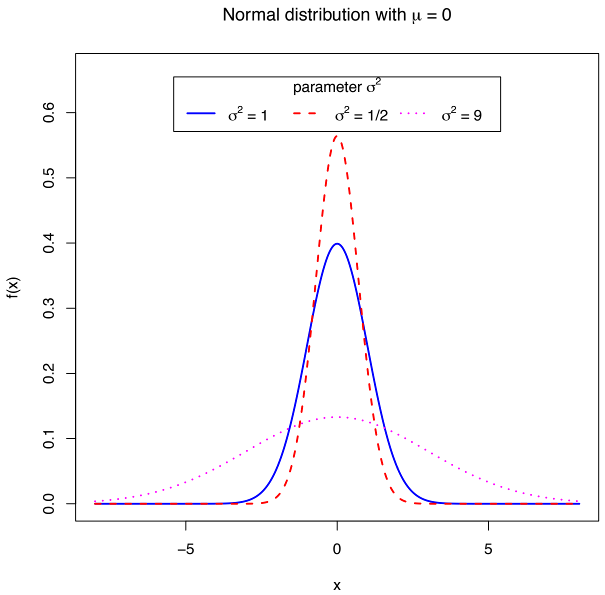

Normal Distribution

Properties:

- The $Z \sim N(0,1)$ is called the standard Normal distribution and for any other Normal, $X \sim N\left(\mu, \sigma^2\right)$ we have:

$$Z = \frac{X-\mu}{\sigma} \sim N(0,1) \qquad \text{and} \qquad X = \mu + \sigma Z$$

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- The Normal is well known for its $68 - 95 - 99.7$ rule, i.e.:

$$\begin{aligned} P(|X-\mu| \leq \sigma) &= P(|Z| \leq 1) &= 0.6826 \\ P(|X-\mu| \leq 2\sigma) &= P(|Z| \leq 2) &= 0.9544 \\ P(|X-\mu| \leq 3\sigma) &= P(|Z| \leq 3) &= 0.9973 \end{aligned}$$

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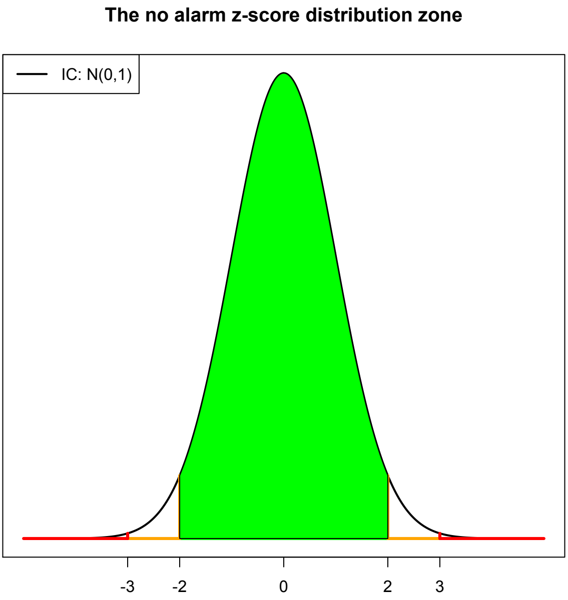
- The above property establishes the well known alarming zones of z-scores (ISO 17043 recommended by the ISO 15189 norm). Specifically:

  Orange alarm: when $-3 <$ z-score ≤ -2 or $2 \leq$ z-score < 3

  Red alarm: when z-score ≤ -3 or z-score ≥ 3

The Alarm Zones



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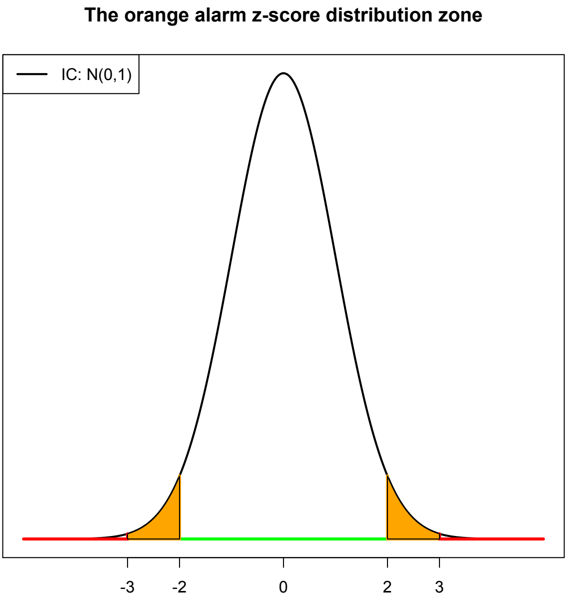

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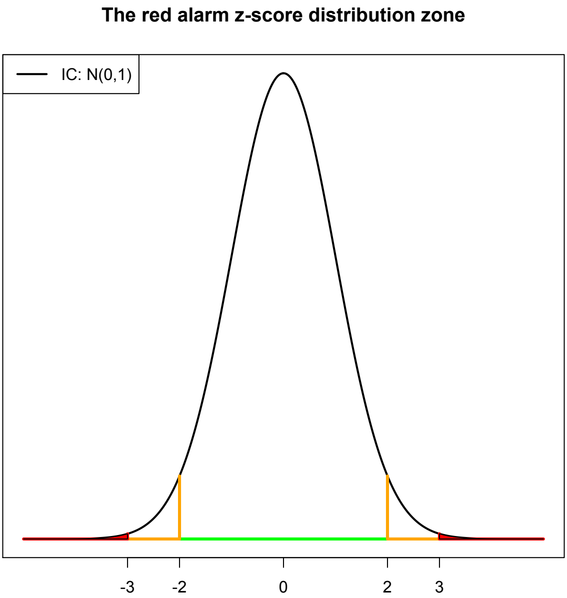

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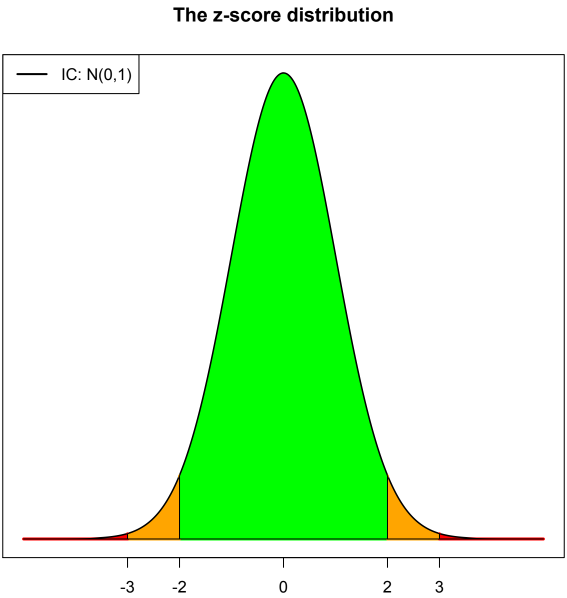
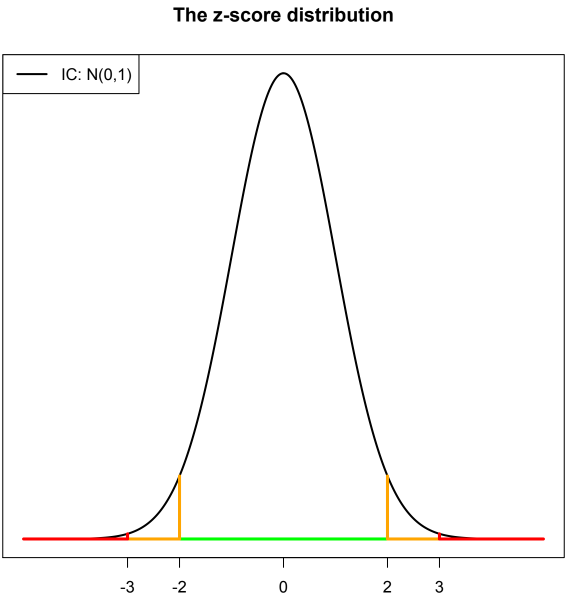


Simulating IC performance



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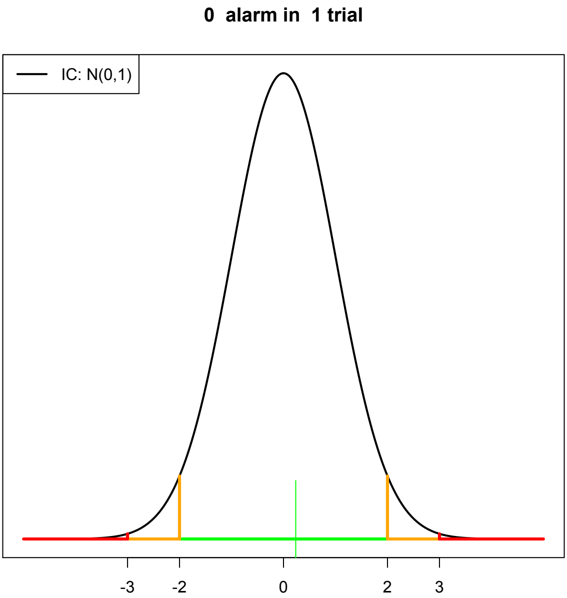

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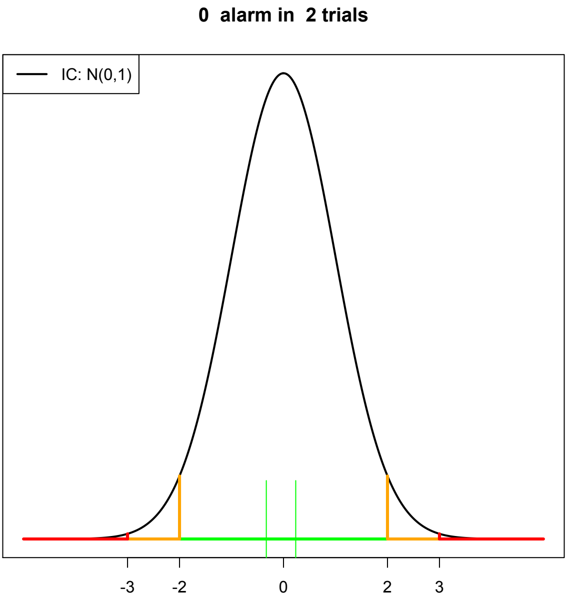

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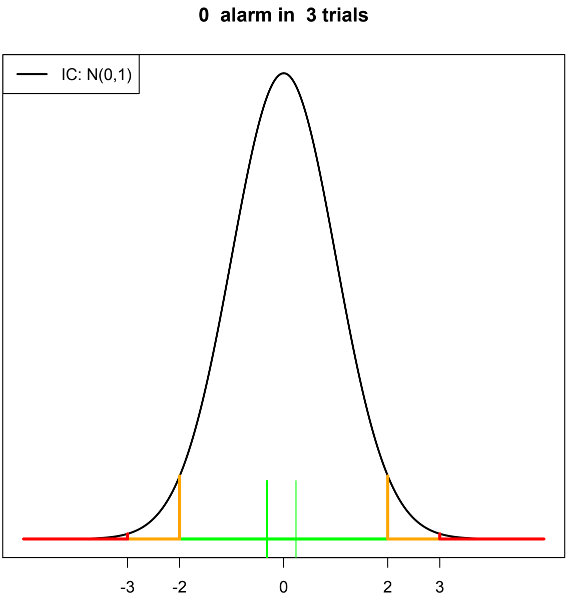

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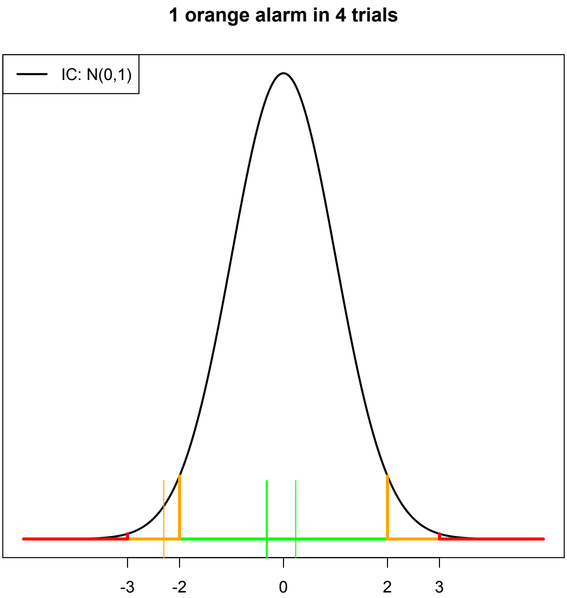

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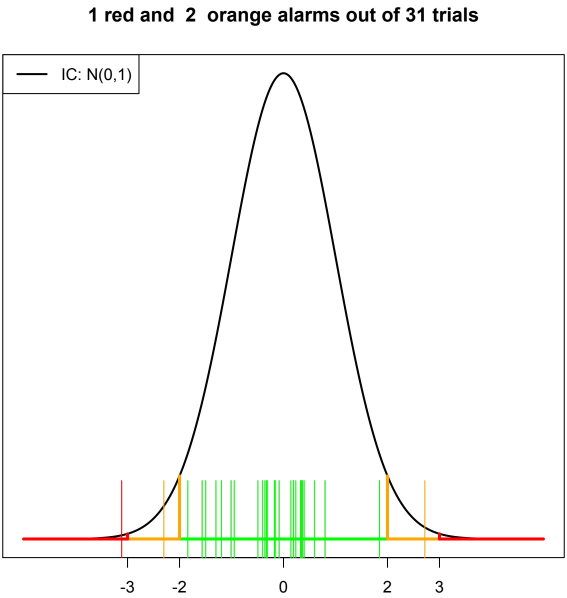

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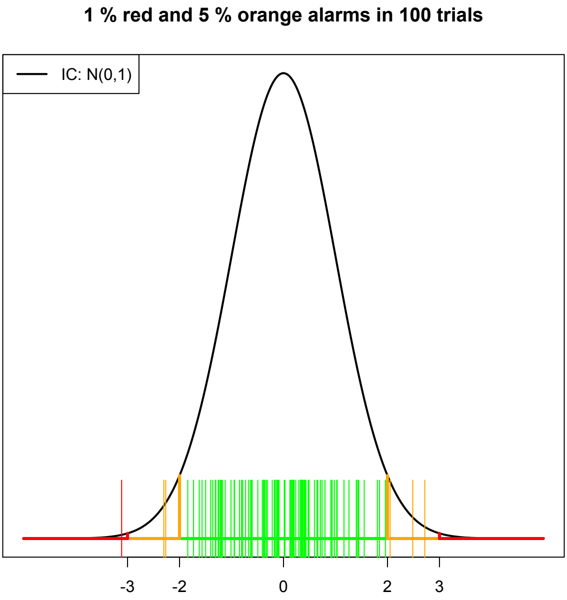

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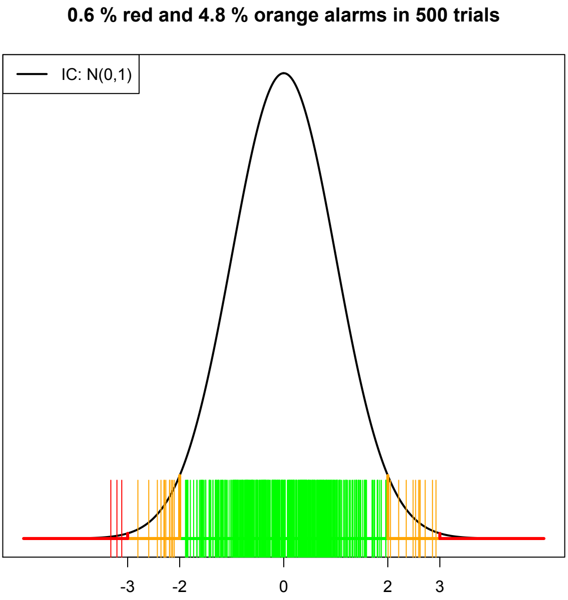

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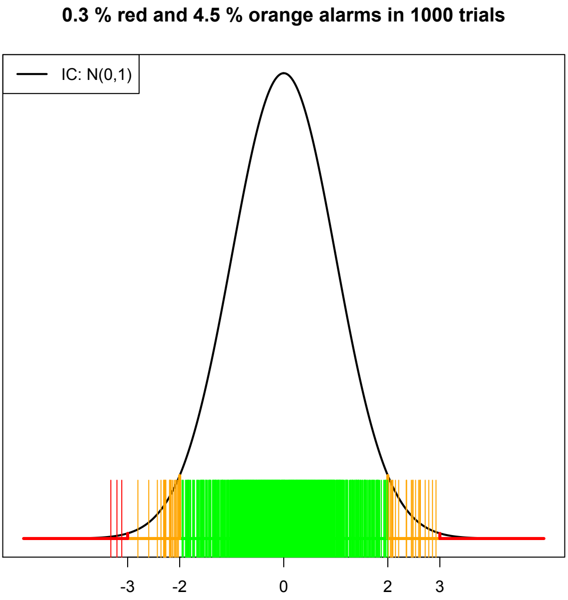

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  - **Uncertainty**: how variable (uncertain) are we?

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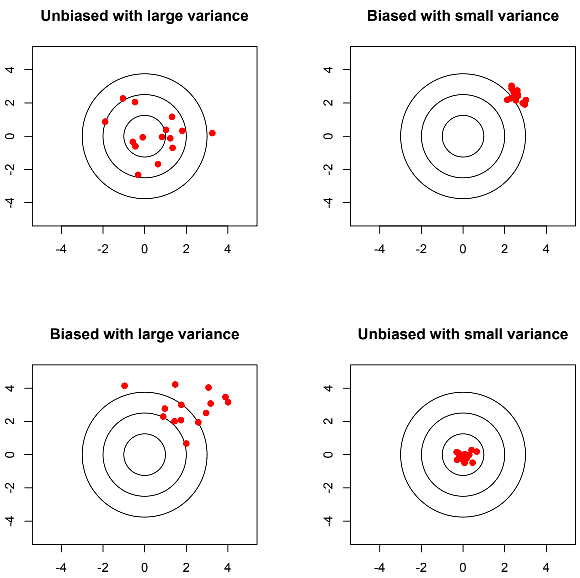

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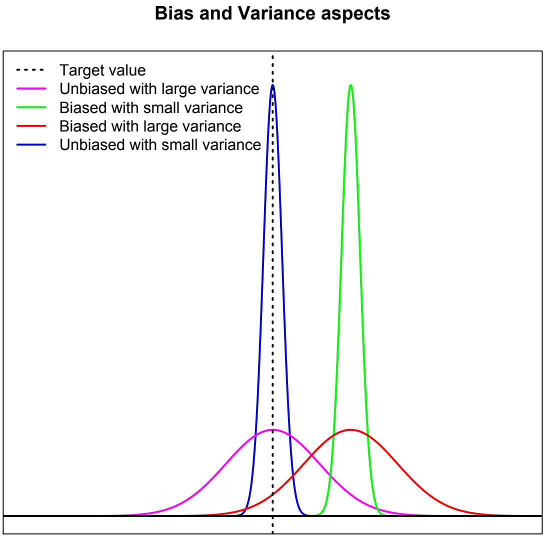

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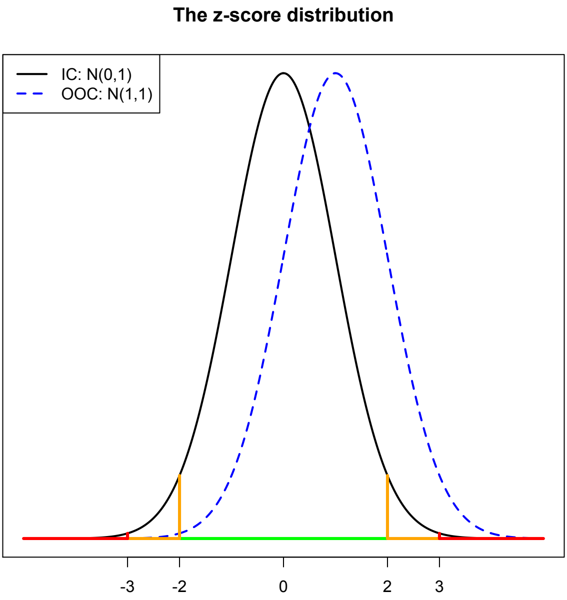

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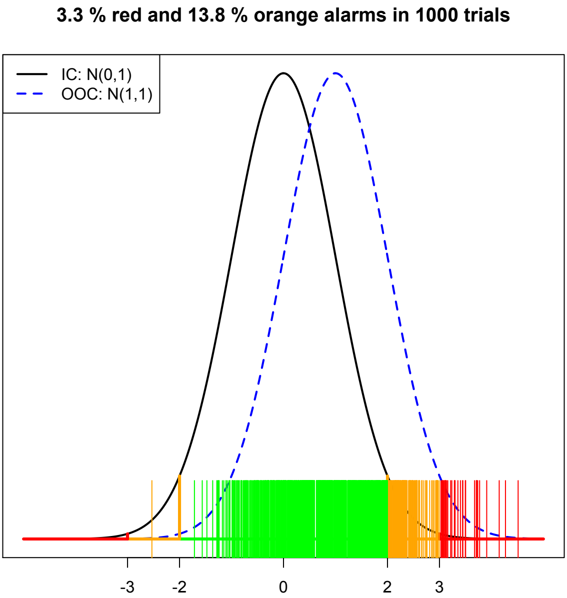

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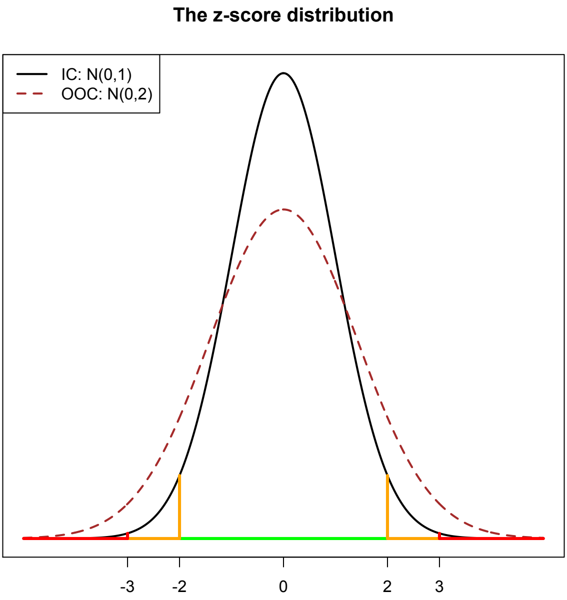

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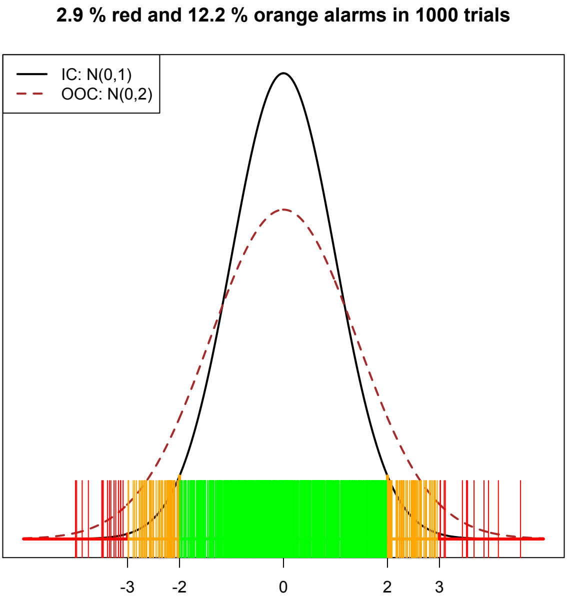

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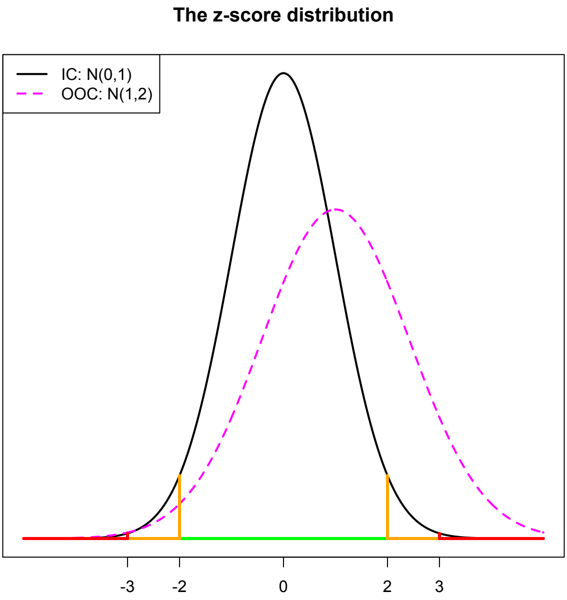

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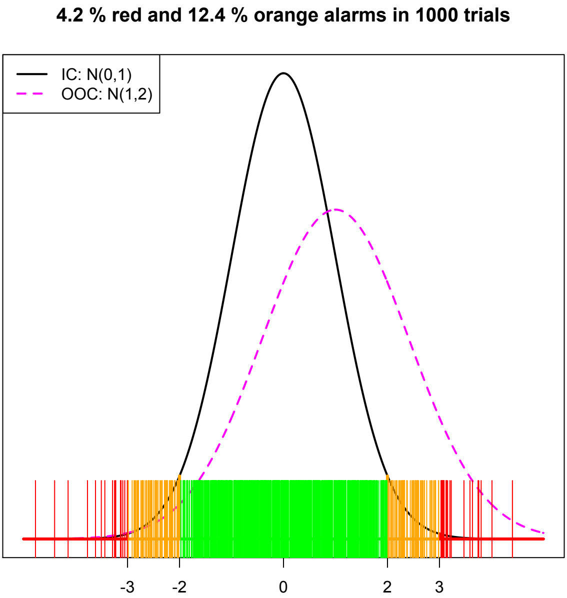

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- If we will consider that the tests are independent and we call Y the number of false alarms that we have in N tests (z-scores of automates), we get $Y \sim B(N, \alpha)$, a Binomial distribution. Then, the probability of at least one false alarm in N tests will be:

$$P(Y \geq 1) = 1 - P(Y = 0) = 1 - (1 - \alpha)^N$$

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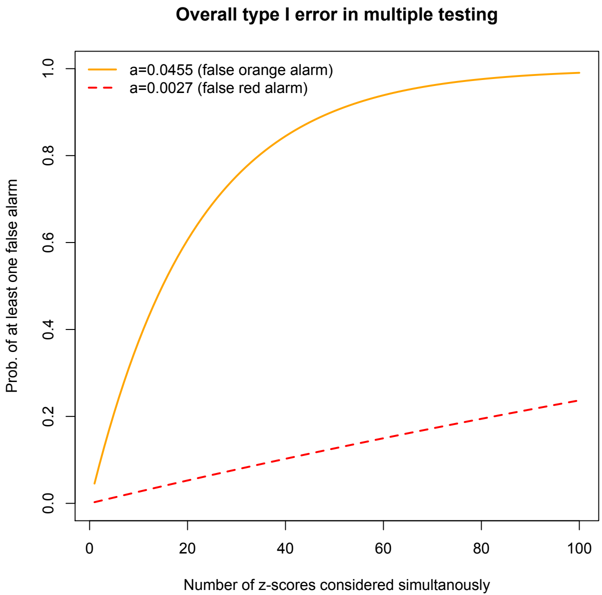

Multiple z-score analysis

The probability of a lab to have at least one orange/red false alarm as a function of the number of automates tested is:

Number of # automates	Orange Zone False Alarm Prob	Red Zone False Alarm Prob
1	4.3%	0.3%
2	8.4%	0.5%
3	12.3%	0.8%
4	16.1%	1.1%
5	19.6%	1.3%
6	23.1%	1.6%
7	26.4%	1.9%
8	29.5%	2.1%
9	32.5%	2.4%
10	35.4%	2.7%

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- So, when we compare two labs with different number of z-scores, how can we adjust the z-score alarm zones, so that the probability of false alarm is approximately the same in both labs?
- In statistics when we perform multiple testing we can adjust the probability of type I error based on the number of tests performed. Assuming independence among the N tests performed, a popular adjustment is the **Bonferroni correction** where it adjusts the type I error by dividing α with the number of tests performed (i.e. α/N).

Bonferroni Correction

Fixing the orange & red based false alarm rates to be always at 0.0428 & 0.0027 respectively, the limits that we need to use, based on the number of z-scores (automates) we test are:

Number of # automates	Orange Zone Limits	Red Zone Limits
1	± 2.00	± 3.00
2	± 2.28	± 3.21
3	± 2.43	± 3.32
4	± 2.53	± 3.40
5	± 2.61	± 3.46
6	± 2.67	± 3.51
7	± 2.72	± 3.55
8	± 2.77	± 3.58
9	± 2.80	± 3.62
10	± 2.84	± 3.64

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- Also we need to pay attention to the evolution in time of the z-scores for each automate, as consecutive alarms in an automate have negligible probability of being actually a false alarm.
- Apart from Bonferroni, other corrections are available in the statistical literature, like:
  - Sidak Correction
  - Holm Bonferroni Correction
  - False Discovery Rate
  - ...

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- If Z_1 is **independent** of Z_2, then:

$$\begin{aligned} P((Z_1, Z_2) \in [-2, 2]^2) &= P(-2 \leq Z_1 \leq 2) \times P(-2 \leq Z_2 \leq 2) \\ &= 0.9545 \times 0.9545 = 0.9110 \end{aligned}$$

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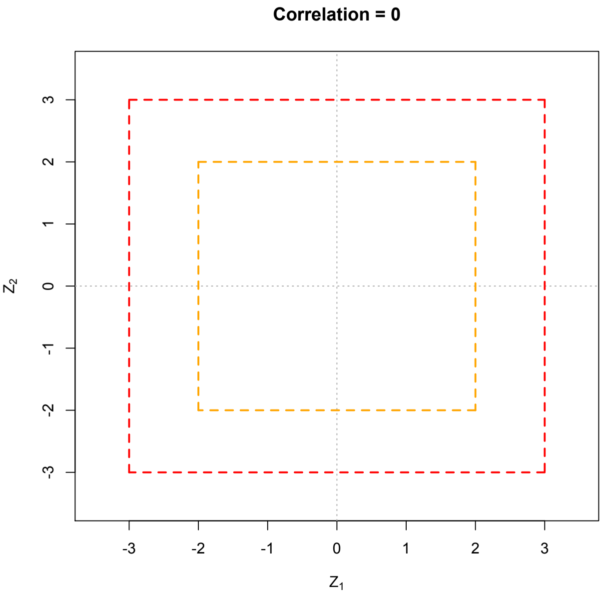
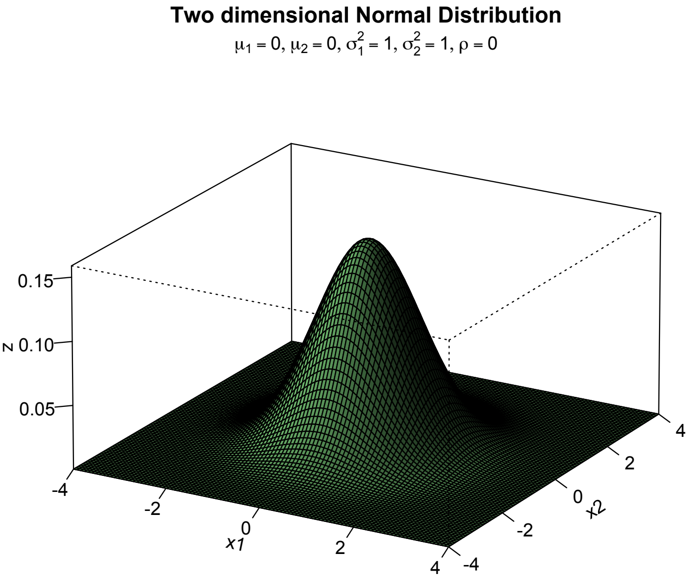
- For correlated variables things become a lot worst when we use the $[-2, 2]^2$ and $[-3, 3]^2$ boxes. One should make use of the Bivariate Normal distribution to model pairs of Z-scores.

Evaluating independent z-scores in pairs



Bivariate Normal distribution (independent)



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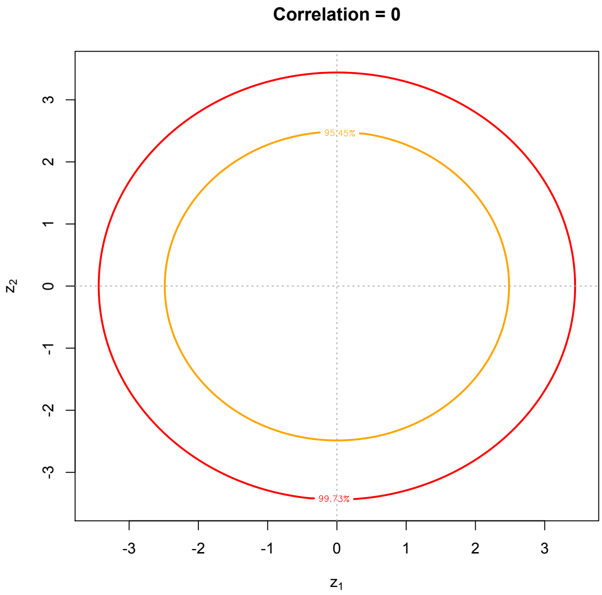

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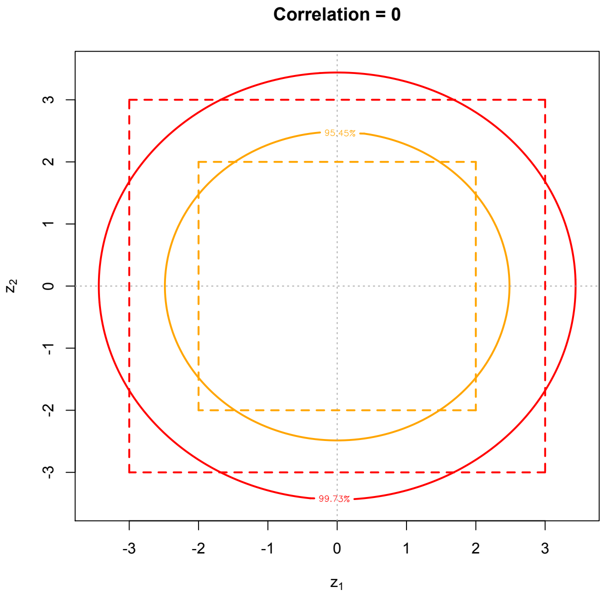

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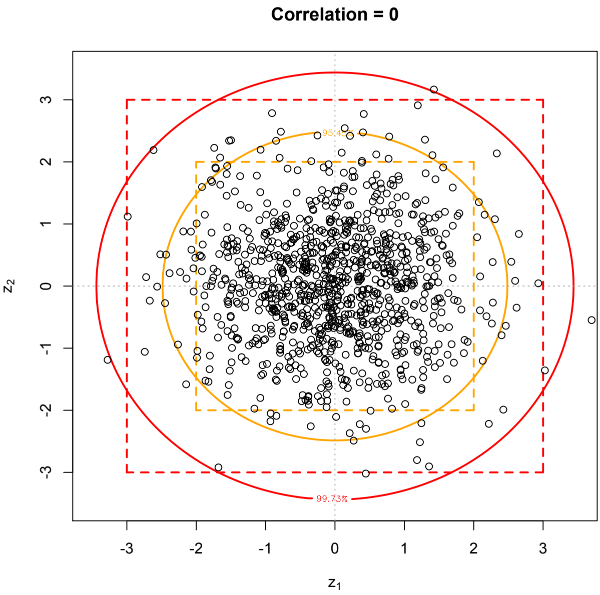

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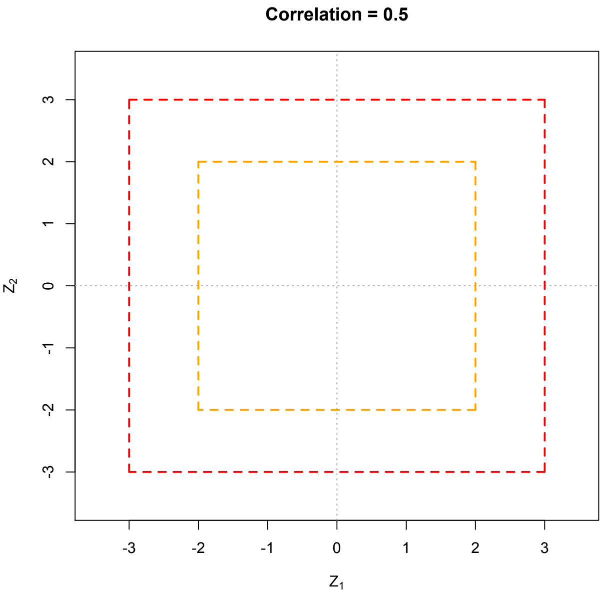
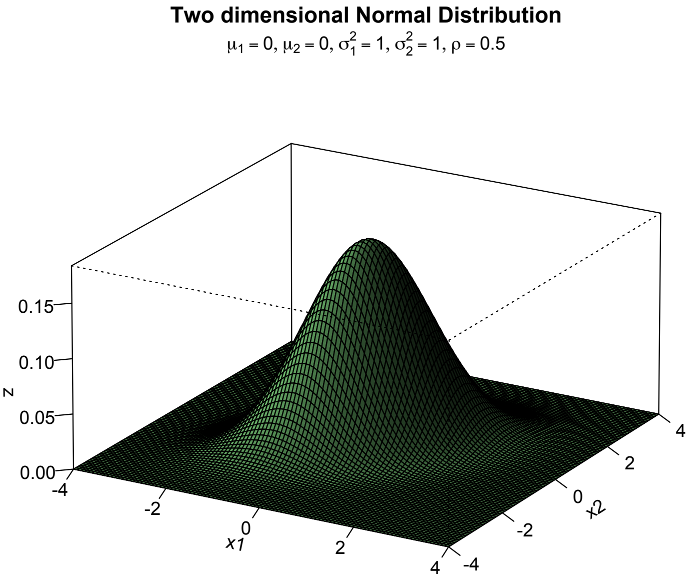


Bivariate Normal distribution (mild positive corr.)



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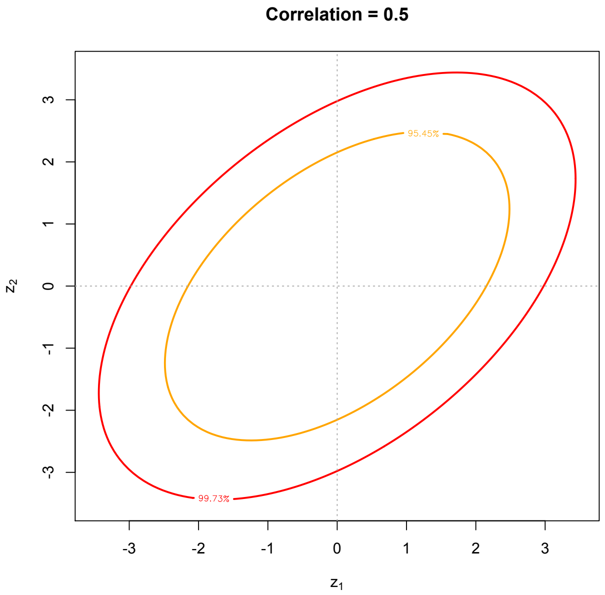

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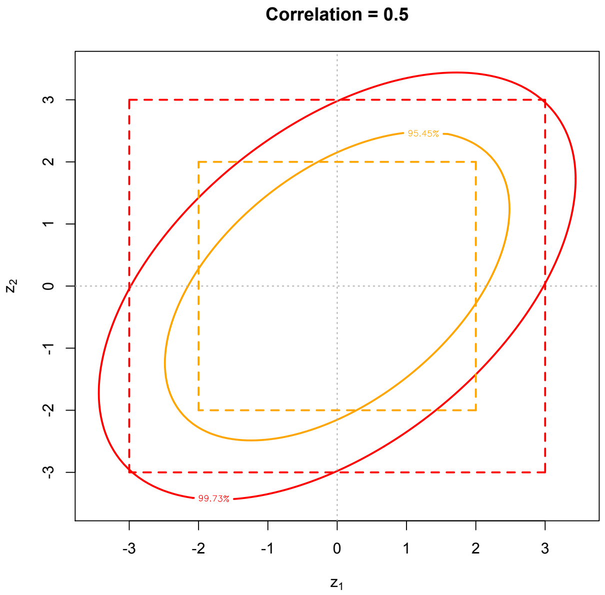

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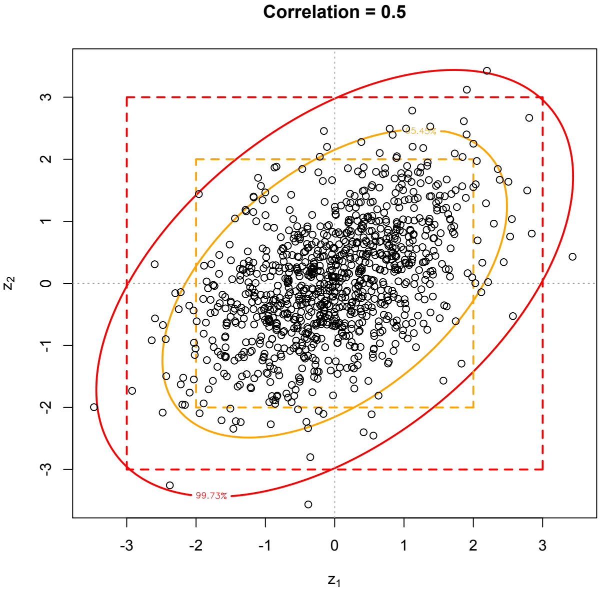

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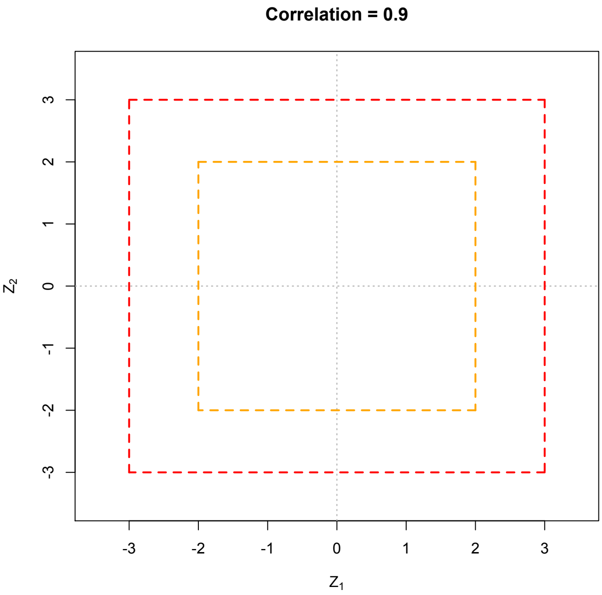
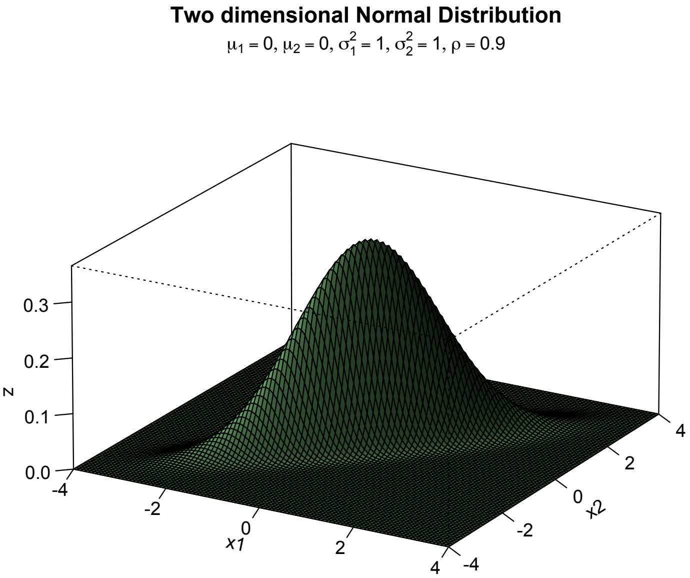


Bivariate Normal distribution (high positive corr.)



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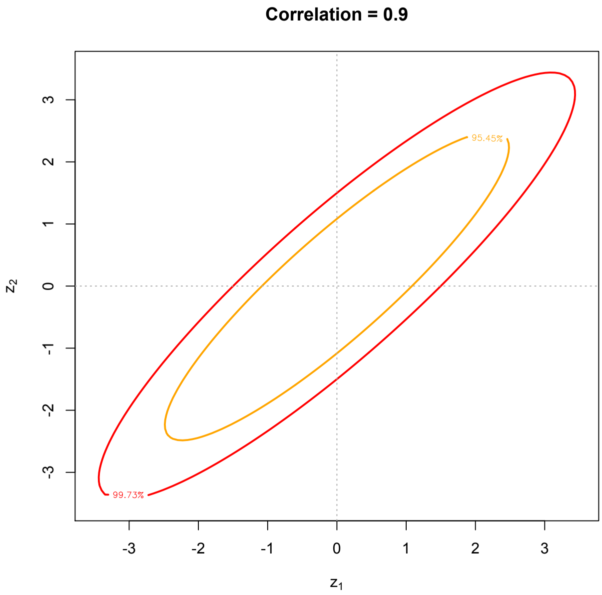

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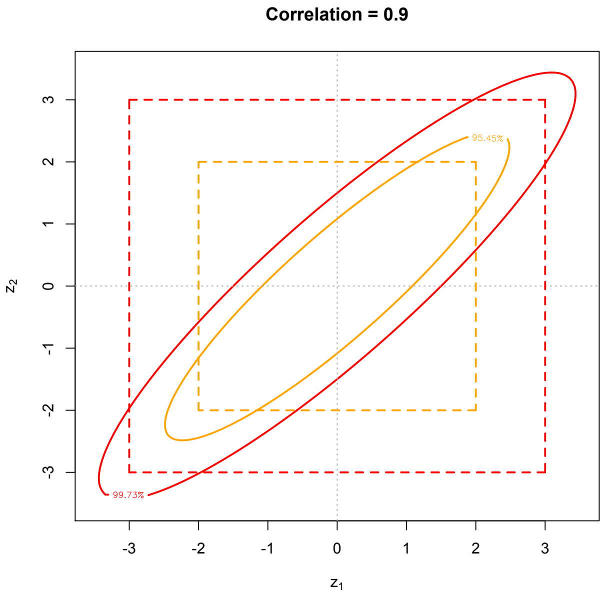
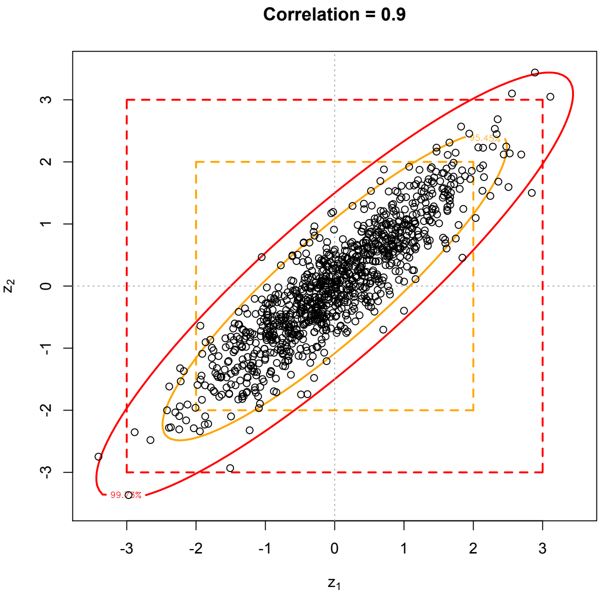


Evaluating correlated z-scores in pairs



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- Bivariate and multivariate analysis methods used in Statistical Process Control (e.g. **Hotelling's** T^2) can be used to detect issues which are not only related to the **magnitude** of the z-score but to the **correlation** as well.

Evaluating z-score history

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- If for each survey we have two results, one can obtain a bivariate plot of the historic z-scores.
- Statistical Process Control tools can be used to detect transient shifts or persistent trends and since we typically have short horizon of data Bayesian methods are expected to be most appropriate.

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- Statistical Process Control tools can be very helpful not only for identifying problems in the ICQ/EQA analysis, but also providing feedback, useful to the root cause analysis.

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The End

Thank you!